Name Solutions

Section: (check one) Jiao ____ Melloch____

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ECE 311 Exam 1 Fall 2008

Closed Text and Notes

1) Be sure you have 10 pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) No calculators

4) Write neatly, if your writing is illegible then print.

5) The last 2 pages contain equations that may be of use to you.

6) This exam is worth 100 points.

(6 pts) 1. Convert the point with Cartesian coordinates of (2, 2, 7) to cylindrical coordinates

$$\rho = \sqrt{x^2+y^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1} 1 = \pi/4$$

$$\left(2\sqrt{2}, \frac{\pi}{4}, 7\right)$$

(6 pts) 2. The intersection of the surfaces defined by r = 10 and $\theta = \pi/4$ is a

- a) a point
- b) a straight line
- [✓ c) a circle]
- d) a plane
- e) a cone
- d) a cylinder

(6 pts). 3. At a point P, the vector force on a 3 C charge is $\mathbf{9a}_\rho - 6\mathbf{a}_\phi - 3\mathbf{a}_z$ N. What is the electric field, **E**, in V/m at P?

$$\vec{E} = \frac{\vec{F}}{Q} = \frac{(9\hat{a}_\rho - 6\hat{a}_\phi - 3\hat{a}_z)\,N}{3C}$$

$$= (3\hat{a}_\rho - 2\hat{a}_\phi - 1\hat{a}_z)\frac{N}{C}$$

$$\vec{E} = (3\hat{a}_\rho - 2\hat{a}_\phi - 1\hat{a}_z)\,\frac{V}{m}$$

(6 pts) 4. If V(x,y,z) = 100xyz, find the electric field intensity.

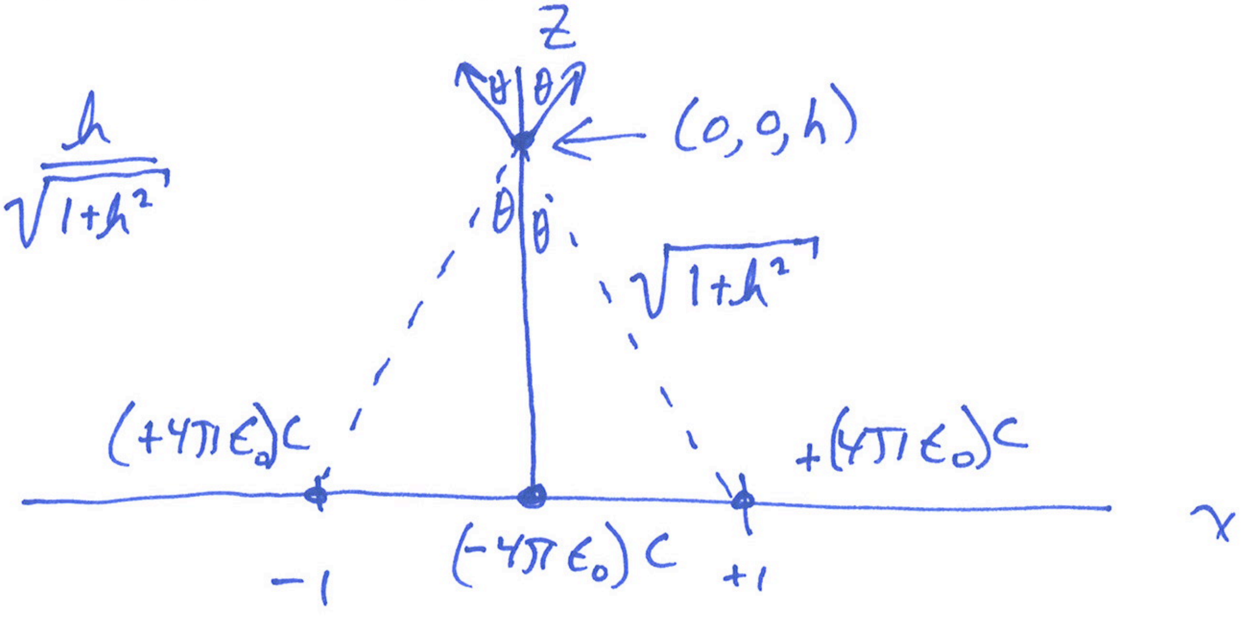
$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{a}_x - \frac{\partial V}{\partial y}\hat{a}_y - \frac{\partial V}{\partial z}\hat{a}_z$$

$$\vec{E} = -100yz\,\hat{a}_x - 100xz\,\hat{a}_y - 100xy\,\hat{a}_z$$

$$= -100(yz\,\hat{a}_x - xz\,\hat{a}_y - xy\,\hat{a}_z)$$

(10 pts) 5. Point charges of $+4\pi\varepsilon_o$ C are placed at (1, 0, 0) and (-1, 0, 0) and a point charge of $-4\pi\varepsilon_o$ C is placed at (0, 0, 0). Determine $\mathbb{E}(0, 0, h)$.

$$\cos\theta = \frac{h}{\sqrt{1+h^2}}$$



There will only be a component of $\vec{E}$ in the $\hat{a}_z$ direction

$$\vec{E}(0,0,h) = \frac{1}{4\pi\epsilon_0}\left[\frac{4\pi\epsilon_0}{1+h^2}\cos\theta + \frac{4\pi\epsilon_0}{1+h^2}\cos\theta - \frac{4\pi\epsilon_0}{h^2}\right]\hat{a}_z$$

$$= \left[\frac{2}{(1+h^2)}\frac{h}{\sqrt{1+h^2}} - \frac{1}{h^2}\right]\hat{a}_z$$

$$\vec{E}(0,0,h) = \left[\frac{2h}{(1+h^2)^{3/2}} - \frac{1}{h^2}\right]\hat{a}_z$$

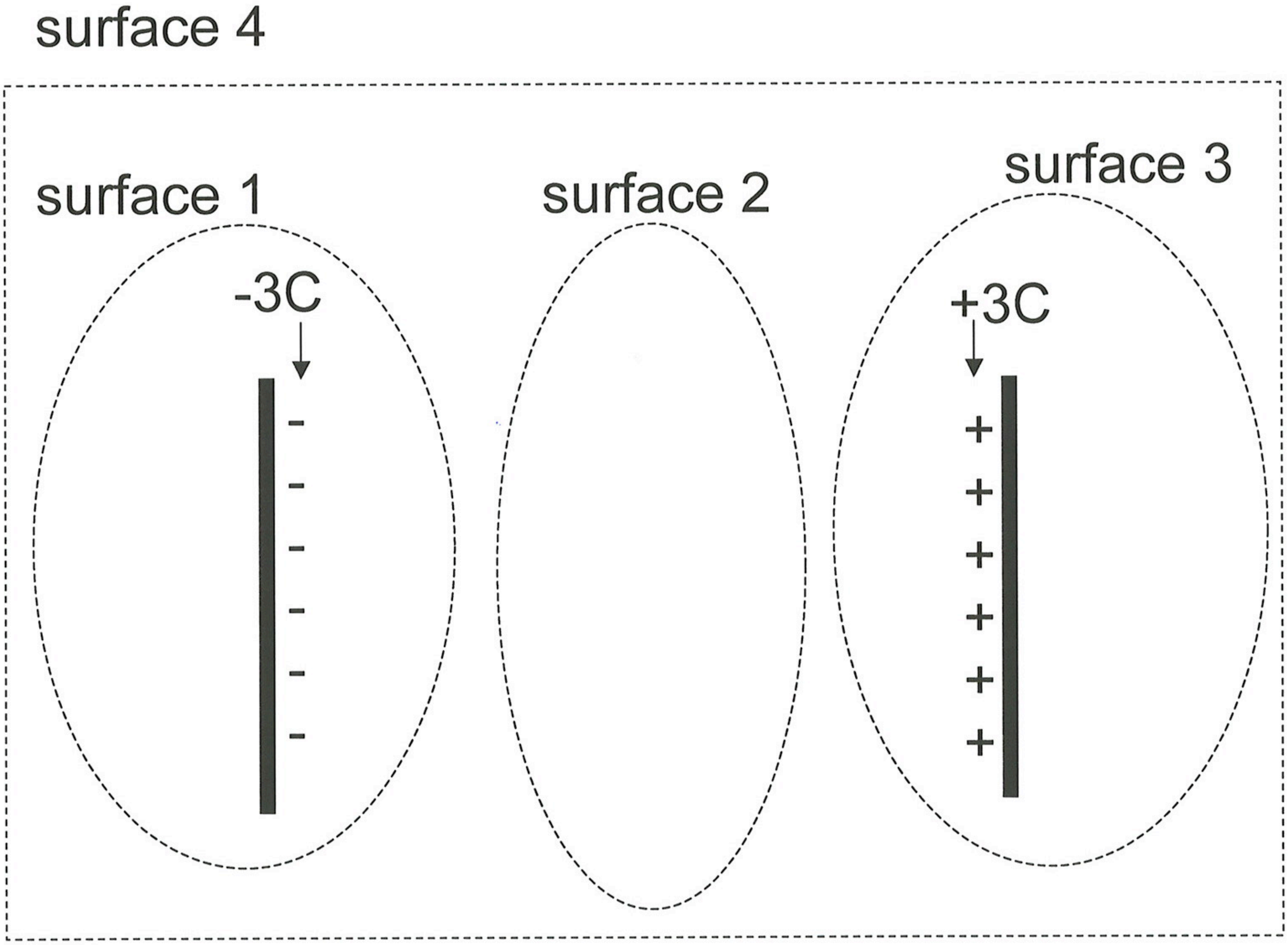
(12 pts) 6. In the following figure the dashed lines represent closed surfaces that completely surround any objects shown within. Determine the following integrals.

$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 1 = $-3\,C$

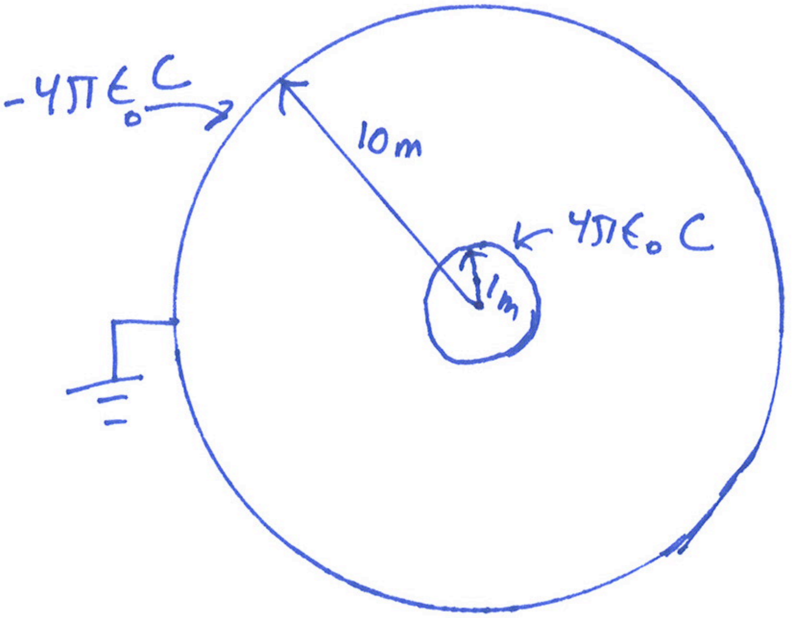
$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 2 = 0

$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 3 = $+3\,C$

$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 4 = 0



(10 pts) 7. A very thin spherical metal shell of radius 1 m is centered at the origin and contains a charge of $+4\pi\varepsilon_o$ C. A very thin spherical metal shell of radius 10 m is centered at the origin and is grounded. Determine the electric field for the three regions, $0 < r < 1$, $1 < r < 10$, and $10 < r$.



The outer shell is grounded, so there is $-4\pi\epsilon_o$ C on the outer shell. So for $r > 10m$ $\vec{E} = 0$. Also, for $r < 1m$ $\vec{E} = 0$.

For $1m < r < 10m$, the $\vec{E}$ field will be the same as if a point charge of $4\pi\epsilon_o$ C is at the origin. So

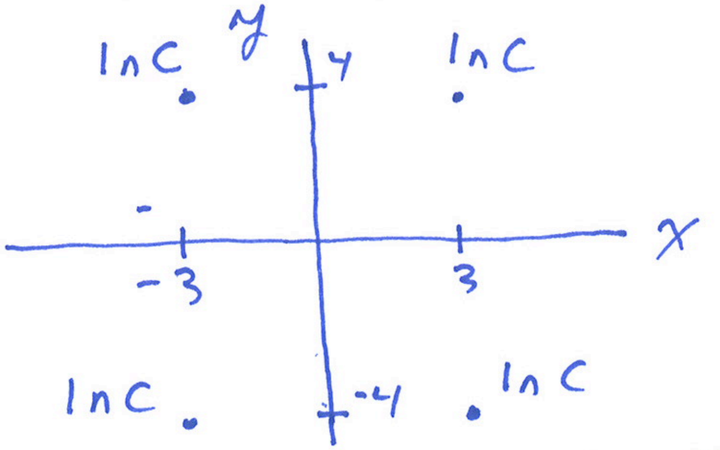
$$\vec{E} = \frac{1}{4\pi\epsilon_o} \frac{4\pi\epsilon_o}{r^2} \hat{a}_r \quad \text{for } 1m < r < 10m$$

so

$$\vec{E} = \begin{cases} 0 & r < 1m \\ \frac{1}{r^2}\hat{a}_r & 1m < r < 10m \\ 0 & r > 10m \end{cases}$$

(20 pts) 8. There are 1 nC charges in the XY-plane at (3, 4, 0), (-3, 4, 0), (3, -4, 0), and (-3, -4, 0).

(10 pts) (a) What is the potential at (0, 0, 0) with V(r=∞) = 0?



each charge is a distance of 5 from (0,0,0). So each charge will contribute the same to the potential at (0,0,0).

$$V(0,0,0) = 4\left[\frac{1}{4\pi\epsilon_0}\,\frac{1nC}{5}\right]$$

$$V(0,0,0) = \frac{10^{-9}}{5\pi\epsilon_0}\ V$$

(10 pts) (b) What is the potential energy of this charge configuration?

$V_k = V_1 = V_2 = V_3 = V_4 =$ the potential at each charge due to the other three charges

$$V_k = \frac{1}{4\pi\epsilon_0}\left[\frac{1nC}{6} + \frac{1nC}{8} + \frac{1nC}{10}\right] = \frac{10^{-9}}{4\pi\epsilon_0}\left(\frac{1}{6} + \frac{1}{8} + \frac{1}{10}\right)$$

$$W_E = \frac{1}{2}\sum_{k=1}^{4} Q_k V_k = \frac{1}{2}(4)Q_1 V_1 = 2\,Q_1 V_1$$

$$= 2\,(10^{-9}c)\,\frac{10^{-9}}{4\pi\epsilon_0}\left(\frac{1}{6} + \frac{1}{8} + \frac{1}{10}\right)$$

$$W_E = \frac{10^{-18}}{2\pi\epsilon_0}\left(\frac{1}{6} + \frac{1}{8} + \frac{1}{10}\right)$$

(20 pts) 9. Circle true or false concerning the following statements.

For a static electric field, $\int_A^B \mathbf{E}\bullet d\mathbf{l}$ is independent of the path from A to B. [✓ true] false

In the spherical coordinate system $\mathbf{a}_\phi$ is always perpendicular to $\mathbf{a}_\theta$ [✓ true] false

The direction of the electric field is always parallel to an equipotential surface. true [✓ false]

The electric flux density on a spherical surface r > b is the same for a point charge Q located at the origin and for charge Q uniformly distributed on a surface r = a where a < b. [✓ true] false

The electric field is a scalar. true [✓ false]

The projection of $2\mathbf{a}_r + \frac{\pi}{3}\mathbf{a}_\theta + \pi\mathbf{a}_\phi$ in the $\mathbf{a}_z$ direction is 1 true [✓ false]

If the divergence of a vector is zero at a point in space, then the vector does not exist at that point, that is the value of the vector is always zero at that point. true [✓ false]

The potential is the energy per unit charge at a point in space. true [✓ false]

A one coulomb charge placed at the origin will create a potential of one volt at the origin. true [✓ false]

If the charge on an object is doubled, the electric flux emanating from that object will drop in half. true [✓ false]

(4 pts) 10. Which one of the following statements best describes the equipotential surfaces surrounding a point charge?

A) The equipotential surfaces are planes extending radially outward from the charge.

B) The equipotential surfaces are curved planes surrounding the charge, but only one passes through the charge.

C) The equipotential surfaces are concentric cubes with the charge at the center.

[✓ D)] The equipotential surfaces are concentric spheres with the charge at the center.

E) The equipotential surfaces are concentric cylinders with the charge on the axis at the center.